Amath Test 5

Find the range of values of k such that the two curves, $y = 3x^2 - 4x + 77$, and $y = 2x^2 + 4k(x + 2) - 11$, will meet. [4]

1.

(a) Find all the angles between $0°$ and $360°$ which satisfy the equation

$$4\sec^2 x = 9\tan x + 13$$

(b) Prove the identity

$$(\tan A - \cot A)\cos A \equiv 2\sin A - cosec\ A.$$

(c) The function h is defined, for $0° \leq x \leq 180°$, by

$$h(x) = 2\sin 3x - 2.$$

(i) State the amplitude and period of h.

(ii) Sketch the graph of $y = h(x)$.

2.

Solve the following equations:

(a) $3^{x+1} - 3 = 2(3^x) + 3^{x-1}$

(b) $\log_9(11x + 21) - \log_9 8 = \log_3(\frac{1}{2}x)$

3.

A rectangle has sides of length $(3 + \sqrt{12})$ metres and $4 + \frac{9}{\sqrt{3}})$ metres.

Express in the form $a + b\sqrt{3}$, where a and b are integers,

(i) the value of A, where A square metres is the area of the rectangle;

(ii) the value of D^2, where D metres is the length of the diagonal of the rectangle.

4.

(a) Given that $f(x) \equiv 2x^3 + px^2 + qx - 6$, where p and q are constants, is exactly divisible by $x - 2$ and leaves a remainder of -10 when divided by $x - 1$.

(i) Find the value of p and of q. [4]

(ii) With these values of p and q, solve the equation $f(x) = 0$. [4]

(b) Given that $2x^3 + 3x^2 - 5x - 9 \equiv (Ax - 3)(x + B)(x + 1) + C$ for all values of x, find the value of each of A, B and C. [4]

5.

Answers

$3x^2 - 4x + 77 = 2x^2 + 4k(x + 2) - 11$

$x^2 + (-4k - 4)x + (88 - 8k) = 0$

Since the two curves will meet,

$D \geq 0$

$(-4k - 4)^2 - 4(1)(88 - 8k) \geq 0$

$16k^2 + 32k + 16 - 352 + 32k \geq 0$

$k^2 + 4k - 21 \geq 0$

$(k + 7)(k - 3) \geq 0$

$k \leq -7, k \geq 3$

1.

(a) $4\sec^2 x = 9\tan x + 13$

$4(1 + \tan^2 x) = 9\tan x + 13$

$4 + 4\tan^2 x = 9\tan x + 13$

$4\tan^2 x - 9\tan x - 9 = 0$

$(4\tan x + 3)(\tan x - 3) = 0$

$\tan x = -\frac{3}{4}$ or $\tan x = 3$

basic angle $= \tan^{-1}\left(\frac{3}{4}\right)$ | basic angle $= \tan^{-1}(3)$

$= 36.87°$ (corr to 1 d.p.) | $= 71.57°$ (corr to 2 d.p.)

$x = 180 - 36.87, 360 - 36.87$ | $x = 71.57, 180 + 71.57$

$x = 71.6°, 143.1°, 251.6°, 323.1°$ (correct to 1 d.p.)

2a.

$$(\tan A - \cot A)\cos A \equiv \left(\left(\frac{\sin A}{\cos A}\right) - \left(\frac{\cos A}{\sin A}\right)\right)\cos A$$

$$\equiv \sin A - \frac{\cos^2 A}{\sin A}$$

$$\equiv \sin A - \frac{1-\sin^2 A}{\sin A}$$

$$\equiv \sin A + \sin A - \frac{1}{\sin A}$$

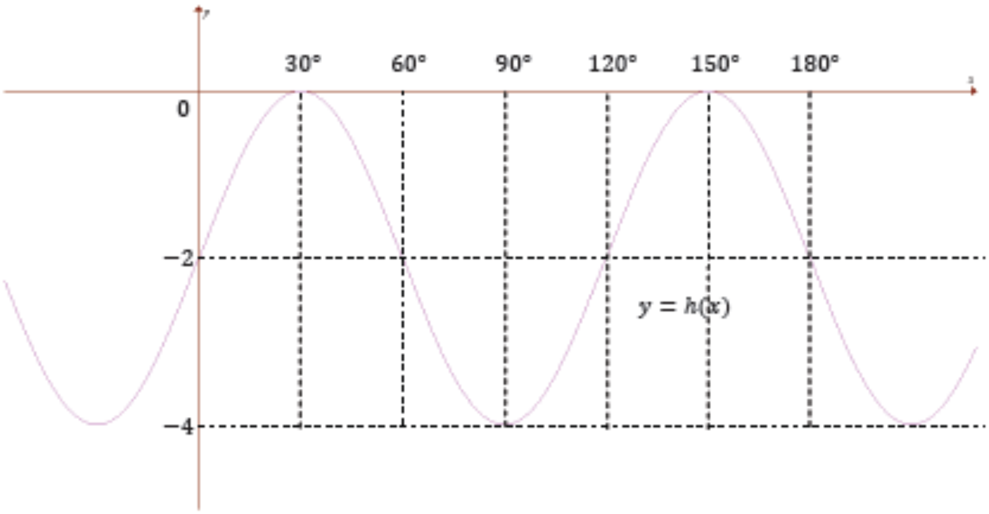
$$\equiv 2\sin A - cosec\ A \text{ (proven)}$$

2b.

(c) (i) Amplitude= 2

Period= 120°

(ii)



2c.

(a) Let $y = 3^x$.

$3y - 3 = 2y + \frac{y}{3}$

$9y - 9 = 6y + y$

$2y = 9$

$y = \frac{9}{2}$

$3^x = \frac{9}{2}$

$x \log 3 = \log \frac{9}{2}$

$x = \frac{\left(\log \frac{9}{2}\right)}{\log 3}$

$= 1.37$ (correct to 3 sig. fig.)

(b) $\log_9(11x + 21) - \log_9 8 = \log_3 \left(\frac{1}{2}x\right)$

$\log_9(11x + 21) - \log_9 8 = \frac{\log_9 \frac{1}{2}x}{\log_9 3}$

$\frac{(11x+21)}{16} = \frac{1}{2}x$

$11x + 21 = 8x$

$3x = 21$

$x = 7$

3.

(a) $A = (3+\sqrt{12})\left(4+\frac{9}{\sqrt{3}}\right)$

$= 12 + 9\sqrt{3} + 4\sqrt{12} + 9\sqrt{4}$

$= 12 + 9\sqrt{3} + 8\sqrt{3} + 18$

$= 30 + 17\sqrt{3}$

(b) $D^2 = (3+\sqrt{12})^2 + \left(4+\frac{9}{\sqrt{3}}\right)^2$

$= 9 + 6\sqrt{12} + 12 + 16 + \frac{72}{\sqrt{3}} + \frac{81}{3}$

$= 37 + 12\sqrt{3} + \frac{72\sqrt{3}}{3} + 27$

$= 64 + 36\sqrt{3}$

4.

(a) (i) $f(2) = 0$

$16 + 4p + 2q - 6 = 0$

$4p + 2q + 10 = 0$

$q = -5 - 2p$ ----- 1

$f(1) = -10$

$2 + p + q - 6 = -10$

$p + q = -6$ ----- 2

Sub 1 into 2

$p - 5 - 2p = -6$

$p = 1$

$q = -5 - 2(1)$

$= -7$

(ii) $f(x) \equiv 2x^3 + x^2 - 7x - 6$

$f(x) = 0$

$2x^3 + x^2 - 7x - 6 = 0$

$(x-2)(2x^2 + 5x + 3) = 0$

$(x-2)(2x+3)(x+1) = 0$

$x = 2, -\frac{3}{2}, -1$

5.

(b) Let $g(x) \equiv 2x^3 + 3x^2 - 5x - 9 \equiv (Ax - 3)(x + B)(x + 1) + C$

$$\equiv Ax^3 + (AB + A - 3)x^2 - (3B - AB + 3)x - (3B - C)$$

By comparing coefficients of x^3,

$A = 2$

By comparing coefficients of x^2,

$3B - 2B + 3 = 5$

$B = 2$

By comparing coefficients of x^0,

$3B - C = 9$

$C = -3$