Marking scheme Paper 2

1. $(-i+\lambda j)\cdot(3i+4j) = \sqrt{1+\lambda^2}\sqrt{3^2+4^2}\cos\theta$

$\boxed{B1}$ $\boxed{B1}$

$-3+4\lambda = \sqrt{1+\lambda^2}\ \sqrt{25}\left(\frac{1}{\sqrt{5}}\right)$ $\boxed{M1}$

$11\lambda^2 - 24\lambda + 4 = 0$ $\boxed{A1}$

$\left.\begin{array}{l}\lambda = 2\\ \lambda = 2/11 \text{ (rejected because } \cos\theta \text{ is positive)}\end{array}\right\}$ $\boxed{A1}$

2. $\dfrac{2\cos 3\theta\sin\theta - 2\cos 5\theta\sin\theta}{2\sin 3\theta\cos\theta + 2\sin 5\theta\cos\theta}$ $\boxed{\text{B1 (using factor formulae)}}$

$= \dfrac{2\sin\theta\sin 4\theta\sin\theta}{2\cos\theta\sin 4\theta\cos\theta}$ $\boxed{\text{B1 (using factor formulae)}}$

$= \tan^2\theta$ $\boxed{\text{A1 (without mistake)}}$

$\tan^2 15^\circ = \dfrac{2\left(\frac{\sqrt{3}}{2}\right)-1-\frac{1}{2}}{2\left(\frac{\sqrt{3}}{2}\right)+1+\frac{1}{2}}$ $\boxed{M1}$

$= \dfrac{2\sqrt{3}-3}{2\sqrt{3+3}}$ or $7-4\sqrt{3}$ $\boxed{A1}$

3. $\sin x\left(\dfrac{\sin x\dfrac{dy}{dx} - y\cos x}{\sin x}\right)$ $\boxed{M1}$

$= \dfrac{dy}{dx} - y\cot x$ $\boxed{A1}$

$\sin x\ \dfrac{d}{dx}\left(\dfrac{y}{\sin x}\right) = \sin x$ $\boxed{B1}$

$\int d\left(\dfrac{y}{\sin x}\right) = \int dx$ $\boxed{B1}$

$\dfrac{y}{\sin x} = x + c$ $\boxed{M1}$

$\dfrac{\pi}{1} = \dfrac{\pi}{2} + c$ $\boxed{M1}$

$y = \left(x + \dfrac{\pi}{2}\right)\sin x$ $\boxed{A1}$

4a) $\int \dfrac{1}{10k - v}\,dv = \int \dfrac{1}{k}\,dt$ $\boxed{B1}$

$-\ln(10k - v) = \dfrac{t}{k} + c$ $\boxed{M1}$

$-\ln(10k - 0) = \dfrac{0}{k} + c$ $\boxed{\text{M1 (subs.t \& find c)}}$

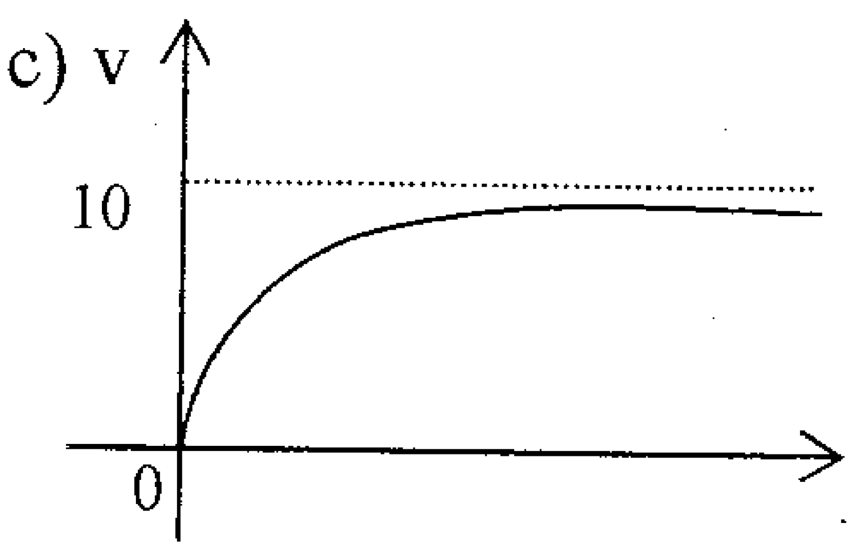
$\left.\begin{array}{l}\ln\left(\dfrac{10k}{10k-v}\right) = \dfrac{t}{k}\\ \dfrac{10k}{10k-v} = e^{-\frac{t}{k}}\end{array}\right\}$ $\boxed{M1}$

$v = 10\left(k - e^{-\frac{t}{k}}\right)$ $\boxed{A1}$

b) $t \to \infty,\ e^{-\frac{t}{k}} \to 0,\ v \to 10k$ $\boxed{M1}$

$v = 10k$ (constant) $\boxed{A1}$

c)



$\boxed{\begin{array}{l}\text{D1 (basic shape cor.)}\\ \text{D1 (all cor.)}\end{array}}$

5.a) OA = OB (radii) $\boxed{B1}$

$\angle$ OAT = $\angle$ OBT = 90°

(tangent perpendicular to radius) $\boxed{B1}$

$\left.\begin{array}{l}\text{OT = OT (common side)}\\ \therefore\ \Delta\text{ OAT} \equiv \Delta\text{ OBT (RHS)}\end{array}\right\}$ $\boxed{B1}$

$\Rightarrow$ TA = TB $\boxed{B1}$

b) In Δ WTX and YTZ

$\angle$ WTX = $\angle$ YTZ (common $\angle$) $\boxed{B1}$

$\angle$ WXT = $\angle$ YZT ($\angle$ in the same segment) $\boxed{B1}$

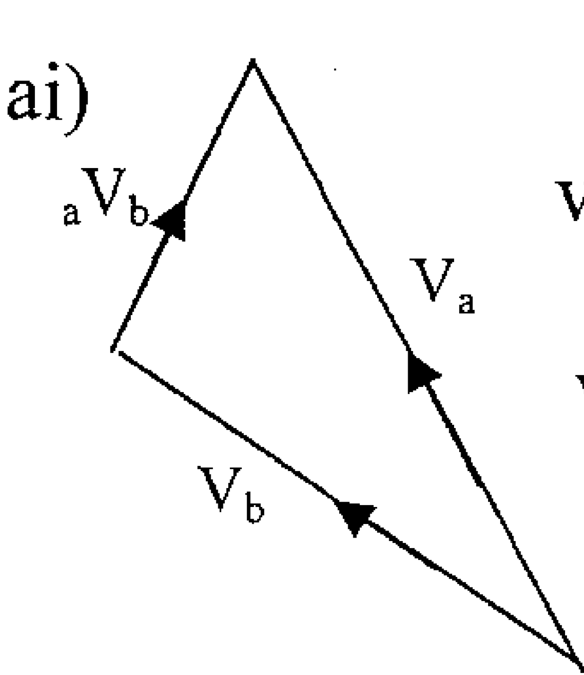
$\angle$ XWT = $\angle$ ZYT (sum of $\angle$s in Δ) $\boxed{B1}$

Δ WTX and YTZ are similar $\boxed{B1}$

$\therefore\ \dfrac{TX}{TZ} = \dfrac{WT}{YT}$ $\boxed{B1}$

$TX \cdot TY = TZ \cdot TW$ $\boxed{B1}$

6 ai)



$v^2 = 20^2 + 15^2 - 2(20)(25)\cos 35^\circ$

$= 133.5^\circ.$ $\boxed{M1}$

$v = 11.55\ \text{kmh}^{-1}$

or $11.6\ \text{kmh}^{-1}$ $\boxed{A1}$

$\boxed{\begin{array}{l}\text{R1 (arrow } V_a \text{ and } V_b \text{ cor. and } \Delta \text{ cor.}\\ \text{R1 (all cor.)}\end{array}}$

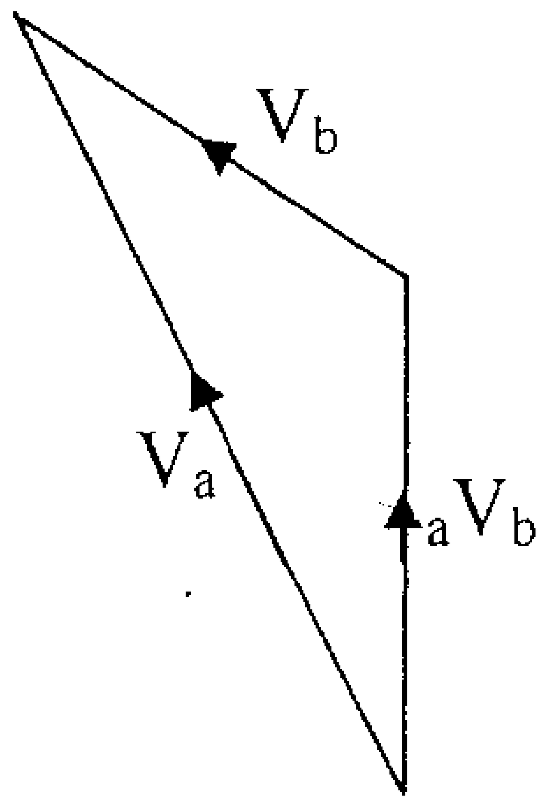
$\dfrac{\sin\theta}{15} = \dfrac{\sin 35}{11.55}$ M1

$\theta = 96.7^\circ$

Direction – N180° -60° - 96.7° E

= N23.3°E or N23.2°E A1

6aii)



$\dfrac{\sin\alpha}{15} = \dfrac{\sin 120}{20}$ M1

$\alpha = 40.5^\circ$

course – N 40.5° W A1

R1 (all cor.)

6b) $\overrightarrow{OA}/\underline{\mathbf{r}}_A = 20t\,i + (50 + 10t)\,j$ B1

$\overrightarrow{OB} = (80 - 10t)\,i + (20 + 30t)\,j.$ B1

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (80 - 30t)\,i + (-30 + 20t)\,j$ M1

$t = 2,\quad \overrightarrow{AB} = 20\,i + 10\,j$ M1

$|\overrightarrow{AB}| = \sqrt{20^2 + 10^2}$ M1

$= 10\sqrt{5}$ km A1

7. a) $\dfrac{P(A' \cap B)}{P(B)} = \dfrac{3}{5}$

$\dfrac{P(B) - P(A \cap B)}{P(B)} = \dfrac{3}{5}$ B1

$5\,P(B) - 1/3 = 3P(B)$ M1

$P(B) = \dfrac{1}{6}$ A1

b) $P(A)P(B) = \dfrac{2}{5} \times \dfrac{1}{6} = \dfrac{1}{15} = P(A \cap B)$ B1

since $P(A)\,P(B) = P(A \cap B)$ → must state

A & B are indep. A1

8. i) $25 = E(X^2) - (-5)^2$ M1

$E(X^2) = 50$ A1

ii) Var (3X – 4Y – 6)

= 9Var(X) +16Var(Y) M1

$= 9(25) + 16[73 - 8^2]$, M1 (+ sign)

= 369 A1

9. i) $\dfrac{2e^{-\lambda}\lambda^1}{1!} = \dfrac{e^{-\lambda}\lambda^2}{2!}$ M1

$4\lambda = \lambda^2$

$\lambda \neq 0,\ \lambda = 4$ A1

ii) $P(X > 3) = 1 - P(X{=}0) - P(X{=}1)$

$- P(X{=}2) - P(X{=}3)$ B1

$= 1 - e^{-4}\left[1 + 4 + \dfrac{4^2}{2!} + \dfrac{4^3}{3!}\right]$ M1

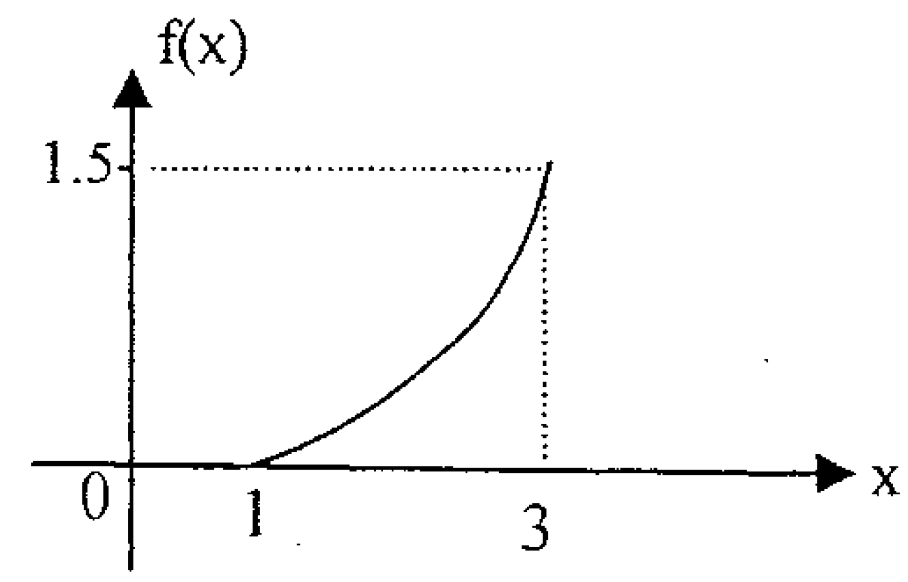
= 0.5665 or 0.567 A1

10. i) $k(3-1)^3 = 1$ M1

$k = 1/8$ A1

ii) $f(x) = \dfrac{dF(x)}{dx} = \begin{cases} \dfrac{3}{8}(x-1)^2, & 1 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}$ A1

A1 (all cor.)



D1

D1 (all cor.)

iii) $E(X) = \displaystyle\int_1^3 \frac{3}{8}x(x-1)^2\,dx$ B1

$= \dfrac{3}{8}\left[\dfrac{x^4}{4} - \dfrac{2x^3}{3} + \dfrac{x^2}{2}\right]_1^3$ M1

$= \dfrac{3}{8}\left[\dfrac{1}{4} - 2(9) + \dfrac{9}{2} - \dfrac{1}{4} + \dfrac{2}{3} - \dfrac{1}{2}\right]$

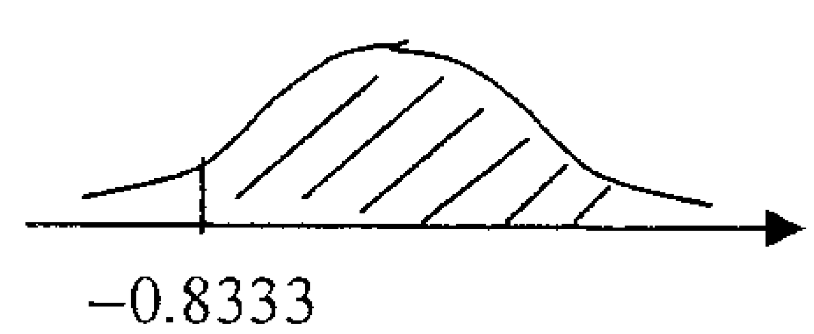
= 5/2 A1

11. $M \sim N(4.1, 0.12^2)$, $D \sim N(5.2, 0.15^2)$

a) $P(M > 4) = P\left(z > \dfrac{4 - 4.1}{0.12}\right)$ B1

$= 1 - P(Z > 0.8333)$ M1

or



= 0.7977, 0.7976, 0.798 A1

b) $X \sim Bin(10, 0.7977)$ $\boxed{\text{B1 (identify Bin.)}}$

$P(X \geqslant 8) = P(X=8) + P(X=9) + P(X=10)$

$= {}^{10}C_8\ (0.2023)^2(0.7977)^8 +$

${}^{10}C_9\ (0.2023)(0.7977)^9 + (0.7977)^{10}$ $\boxed{M1}$

$= 0.6708$ $\boxed{A1}$

11c) Let $T = (M_1 + M_2 + M_3 + M_4 + M_5) - (D_1 + D_2 + D_3 + D_4)$ $\boxed{B1}$

$E(T) = 5(4.1) - 4(5.2)$ $\boxed{M1}$

$= -0.3$

dep

$Var(T) = 5(0.12^2) + 4(0.15^2)$ $\boxed{M1}$

$= 0.162$

$T \sim N(-0.3, 0.162)$

$P(T > 0) = P(Z > \frac{0-(-0.3)}{\sqrt{0.162}})$ $\boxed{M1}$

$= P(Z > 0.7454)$

$= 0.2280$ $\boxed{A1}$

12 i) Median $= 49.5 + \left[\frac{\frac{120}{2} - 56}{40}\right]5$ $\boxed{M1}$ $\boxed{B1}$ $\boxed{A1}$

$= 50$ kg

ii) Mean $= \frac{5970}{120}$ $\boxed{M1}$ $\boxed{A1}$

$= 49.75$ kg

$SD = \sqrt{\frac{3011850}{120} - \left(\frac{5970}{120}\right)^2}$ $\boxed{M1}$ $\boxed{B1}$ $\boxed{A1}$

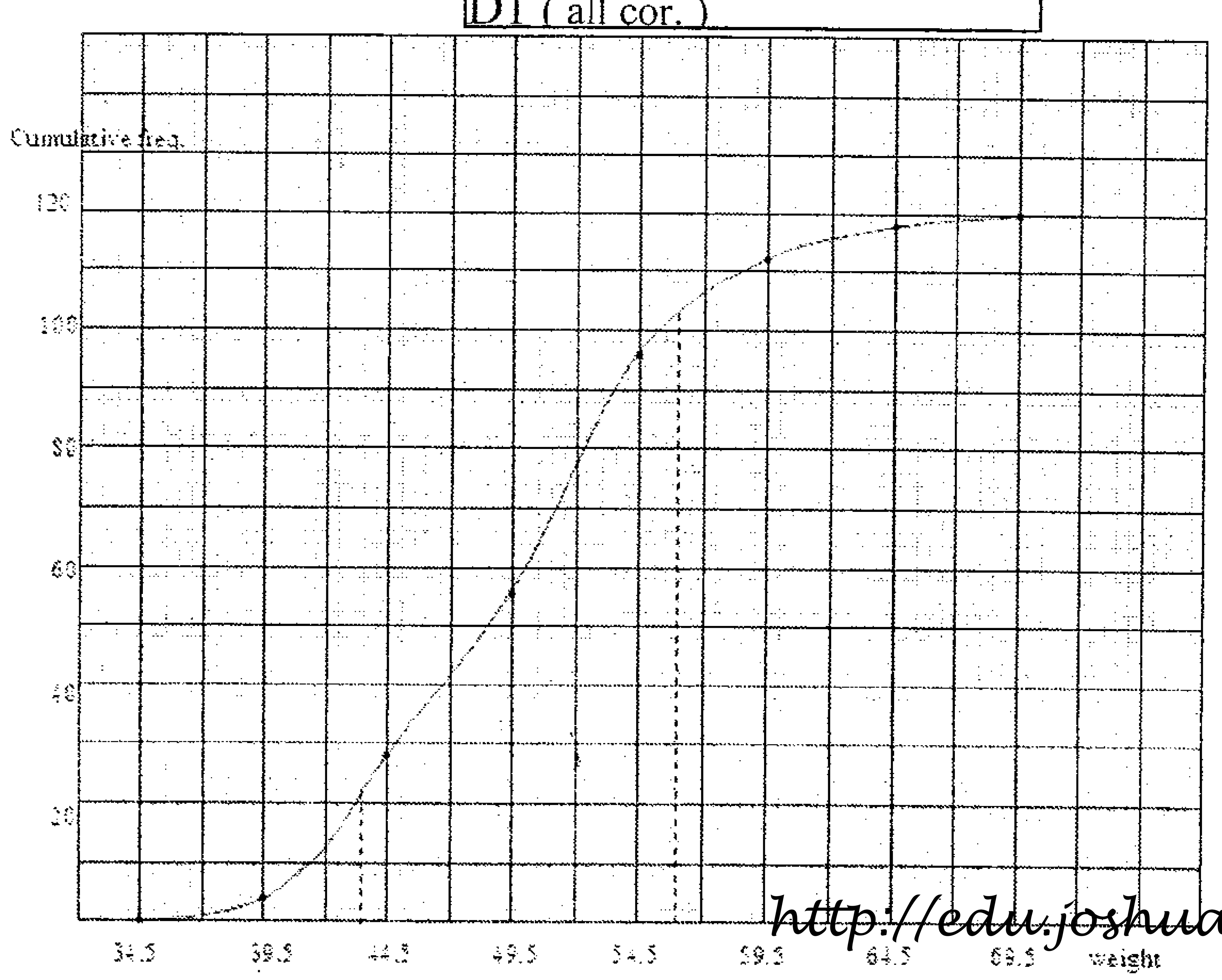
$= 6.35$

iii) Graf

$\boxed{D1}$ (> 6 boundaries cor.)

$\boxed{D1}$ (curve passes through 8 pts.)

$\boxed{D1}$ (all cor.)



iv) Range for 1 sd from the mean = (43.4, 56.1) $\boxed{B1}$

$\% = \frac{103 - 22}{120} \times 100$ $\boxed{M1}$

$= 67.5\ \% \pm 1$ $\boxed{A1}$